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Kappa Distributions in Space Plasmas and Their Role in Suprathermal Particle Dynamics

Sumit Singh
Research Scholar, Department of Physics, Mansarovar Global University,
Sehore, M.P., India.

ABSTRACT

The function of parallel ion flow shear in controlling the behavior of electrostatic ion cyclotron waves in magnetized plasma is confirmed by the current investigation. It is clear from tests that the strongest shear regions are the ones where velocity shear significantly amplifies wave amplitudes. This amplification, which is shown to be a strong shear-driven phenomenon, is consistently observed in both the basic ion cyclotron frequency and higher harmonics. In addition, the results show that the electrons' instabilities in the plasma volume contribute quasilinearly to the wave propagation. Once again, the closeness of the maximum shear and enhanced wave activity areas provides more evidence that flow gradients may be utilized as a fundamental source of plasma instability.

Keywords: Kappa Distribution, Suprathermal, Thermodynamics, Solar Wind, Non-Equilibrium.

I. INTRODUCTION

For decades, scientists have studied space plasmas in a broad variety of astronomical locations and environmental settings. Particles arriving to the device within a specific mass, energy, and angular range are often recorded using instruments that are intended to sample the plasma velocity distribution. The plasma fluid properties, including bulk speed, density, and temperature, are the end goals of many applications that rely on these plasma data. Count rates that account for the attributes of the instruments, which are obtained by ground and/or in-flight calibration methods, are used to produce these higher-level outputs.

Formulating the plasma distribution and then fitting the instrument's response model to the data is a typical way to determine the plasma fluid properties. Alternatively, the velocity moments can be obtained from the observed distribution function using a direct numerical integration technique. The complete distribution function must fit within the energy and angular range of the sensor for this approach to work.

The distribution of plasmas in collisional equilibrium is Maxwellian. But kappa distributions characterize space plasmas in their stationary out-of-equilibrium phases. In a Kappa distribution, the "core" (velocities around the distribution function's bulk velocity) is similar to a Maxwellian distribution, while the high-energy tail follows a power law. In addition to their study in other plasma-related investigations, they have been seen in the solar wind, planetary magnetospheres, the outer heliosphere, and the inner heliosheath. In addition, some research has proven that Tsalis's non extensive statistical mechanics is the natural source of the kappa distributions.

Prior research has demonstrated that substantial misestimations of the plasma temperature can result from fitting a Maxwellian distribution to the center of a kappa distribution. Hence, the kappa distribution must be handled with care, considering the part played by particles with tail velocities. For all values of k from 0 to 1, the limit of the kappa distribution is the Maxwellian distribution. Thus, Maxwellian distributions may be found by using kappa distributions. One other free parameter for the distribution that has to be found is the kappa index, which we have noted. Unfortunately, practical limitations imposed by instruments make it such that high-energy tails are not necessarily visible. One example is the Voyager 2 plasma sensor, which makes it difficult to determine the parameters for the distribution-function fit as it observes a small number of data points within a narrow energy range.

On top of that, the energy range of the device can overlap with two or more plasma populations in certain instances. The high-energy tail of the Jovian magneto sheath proton distribution, for instance, cannot be easily separated from the alpha particle distribution. In these situations, a different way to find kappa distributions is needed, preferably one that can be used with limited range of energy and angles to eliminate, say, distinct particle species.

II. REVIEW OF LITERATURE

Nicolaou, Georgios et al., (2020) Space plasma particle velocities are often distributed according to kappa curves. Plasma dynamics may be better understood with the help of the kappa index, which regulates and labels these distributions. In order to study plasma particles and create velocity distribution functions, space scientific missions frequently include plasma instrumentation. We may find the plasma bulk properties, including density, speed, temperature, and kappa index, by analyzing the velocity distribution functions correctly. The velocity moments of the observed distribution function are often used to calculate the bulk density, velocity, and temperature of the plasma. The kappa index may be computed using the distribution function's speed (kinetic energy) moments, according to current research. This is rather interesting. The results of such an innovative computation may be invaluable in further studies and practical endeavors. The specific method's correctness is investigated in this study by means of synthetic plasma proton measurements taken by a standard electrostatic analyzer. After deriving the plasma bulk parameters from the modeled data, we compare them to the parameters that were originally utilized to simulate the observations. We measure the systematic and statistical mistakes in the derived moments and talk about where they may be coming from by comparing the two.

Nicolaou, Georgios et al., (2018) The kinetic state of space plasmas may be completely described by reconstructing their velocity distributions, which are made possible by measurements made by spacecraft. The kappa distribution is a common tool for characterizing the stationary states of non-equilibrium space plasmas. It is necessary to ascertain the kappa index and temperature, which control these distributions, in order to provide a comprehensive and precise description of these plasmas. Our work presents a new and trustworthy method for calculating the plasma distribution functions' kappa index and temperature using counts recorded in a restricted energy range by conventional electrostatic sensors. Our approach is applicable in situations when the high-energy plasma tail is either not seen or observed with a high degree of ambiguity. To prove that our technique works, we model the configuration of the Solar Orbiter mission's ion plasma sensor SWA-PAS and generate pseudo-observations for common input plasma characteristics. By fitting the angular spread of the distribution in a restricted energy range around the core bulk energy, our technique correctly determines the necessary plasma characteristics. For a solar wind-typical bulk energy, we evaluate our method's output in comparison to input parameters that were utilized to produce synthetic data for a chosen range of kappa index and temperature. We also compare our approach to Helios 2 observations, examine its possible uses and constraints, and investigate the impact of Poisson errors on the counting statistics of the sensor.

Livadiotis, George. (2015) System out of thermal equilibrium, such space plasmas, may be easily replaced by empirical kappa distributions, which offer a more direct alternative to the Maxwell distribution. With the number of relevant articles growing at an exponential rate, Kappa distributions have become increasingly common in space physics. The link between kappa distributions and the nonextensive statistical mechanics paradigm, however, was a game-changer in the area. The mathematical formulations of the theory of kappa distributions, which arise from the coupling of kappa distributions with a strong statistical foundation, are provided in this introductory work along with clarifications of basic physical notions. The study discusses, among other things, the creation of a Hamiltonian's kappa distribution, the physical significance of the kappa index, and the fact that systems out of thermal equilibrium described by kappa distributions have a consistent definition of temperature. Furthermore, the research delves into the most common kappa index values seen in space plasmas. Patterns in the typical density, temperature, and kappa index values of space plasmas are shown by statistical analysis. At last, to construct all the conceivable formulations of isotropic/anisotropic kappa distributions, one must grasp the kinetic interpretation of temperature as the mean kinetic energy and of the kappa index as the correlation of kinetic energies.

Livadiotis, George & Mccomas, David. (2011) Upcoming developments in the field of space physics theory have demonstrated the link between space plasmas and non-extensive statistical mechanics. This is achieved by giving a theoretical foundation to the kappa distributions, which are often employed to explain the phase-space distribution functions of these systems, and which are typically generated from empirical data. Two independent governing factors of non-equilibrium systems are the non-equilibrium temperature and the kappa index, which determine these distributions. The physical meaning of the kappa index is related to the correlation between the system's particles, but its primary duty is to identify the non-equilibrium stationary states and measure their "thermodynamic distance" from thermal equilibrium. A number of non-equilibrium stationary states, denoted by the kappa index, are expanded from the classical, one stationary state at equilibrium. In order to determine stationary states, this work tackles some important questions about the physical significance and function of the kappa index. These discrepancies have arisen because the kappa index is not a constant physical quantity but rather varies on the particle degrees of freedom in the system. Several erroneous implications are drawn from this, including (1) that the many-particle kappa distribution can only be described by an infinitely high kappa index and (2) that the correlation between particles is proportional to the total number of particles in the system. The ability to naturally establish a modified kappa index that remains constant regardless of the number of degrees of freedom is demonstrated here. We proceed to create and analyze the appropriate corrected formalism for many-particle multidimensional kappa distribution, before delving into the physical significance of the invariant kappa index.

Livadiotis, George & Mccomas, David. (2009) In space physics, empirically generated kappa distributions are mixing with more conventional quasi-Maxwellian cores, as the power law structure of different suprathermal tails becomes more common. There are two widely accepted mathematical definitions of kappa distributions, and different writers provide differing descriptions of the power law behavior of suprathermal tails. Our research delves into the spontaneous emergence of kappa distributions from Tsallis statistical mechanics, a theory that offers a firm foundation for the description and analysis of complex systems that are not in equilibrium. The ranges of values that can be assigned to kappa are shown by this analysis. The idea of temperature out of equilibrium, which is very different from traditional equilibrium temperature, is also developed by us. Using this distribution, the kinetic and physical temperatures become one, in and out of equilibrium, and this analysis indicates which of the kappa distributions is superior. Lastly, the link between the two kinds of kappa distributions and the spectral indices often employed to parameterize space plasmas is extracted. By establishing this link, spectral indices from different space physics measurements, models and theoretical investigations involving kappa distributions may be easily and consistently compared, reducing the possibility of misunderstanding and inaccuracy. With the link between Tsallis statistical mechanics and empirically derived kappa distributions finally established, the space physics community can use all the power of Tsallis statistical tools to study and understand the kappa-like features of all the energy and particle distributions seen in space.

Shizgal, Bernie. (2007) We use the Fokker-Planck equation for Coulomb collisions and a quasi-linear diffusion operator for wave-particle interactions to study the energization of a charged test-particle of mass m in equilibrium contact with a large ensemble of charged particles of mass M. This study examines the characteristics of the test-particle system's nonequilibrium steady state velocity distribution as a function of the mass ratio m/M and the respective intensities of the wave-particle interactions and Coulomb collisions. The results demonstrate that a Kappa distribution is not always the case for the stable distribution function. The model predicts that, similar to the solar wind, the temperature of heavy minor ions will change linearly with the mass ratio. The findings obtained are explained by the Kullback relative entropy, not the Tsallis nonextensive entropy, and the temporal evolution of the distribution function with and without energization by wave-particle interactions is computed.

III. DETECTION OF KAPPA DISTRIBUTIONS

In a broad space plasma domain, supermaxwellian velocity distribution functions (VDFs) with suprathermal tails have been seen quite a bit. For such suprathermal populations, kappa distributions with high-energy power-law tails offer a more suitable representation than conventional thermal equilibrium. Several satellite missions have shown that kappa distributions with indices typically 2<K 6 fit particle data well in many interplanetary and near-Earth locations.

Plasmas with a kappa distribution have been found in many different environments, including the solar wind, the magnetosphere of Earth, the radiation belt, the plasma sheet, and the radiation belly. Planets such as Mercury, Jupiter, Saturn, Uranus, and Neptune have magnetospheres that exhibit similar distributions above Earth. Additionally, smaller objects and distinct areas of the plasma, such as Titan's ionosphere and the Io plasma torus, are noted. The frequency of kappa distributions in astrophysical plasmas has been determined with great help from measurements taken by large-scale space missions like Ulysses and Cassini as well as those taken by the Hubble Space Telescope.

In solar wind electron velocity distribution functions, a thermal core, a suprathermal halo and a field-aligned strahl population are often the three main components of a complex structure. The bulk of the electrons at the center of the distribution have low energies, making it essentially a Maxwellian distribution. However, there is an almost isotropic power-law tail of the suprathermal population in the halo component. The strahl element consists of a tiny electron beam that is either perpendicular to the Sun or parallel to the interplanetary magnetic field.

As seen in Figure 1, the distribution of electron velocities in the solar wind is characterized by the thermal core and halo suprathermal populations. Another part of these electron VDF is the strahl, which is in line with the interplanetary magnetic field. Kappa functions were employed by Maksimovic et al. (1997a) to match electron VDF recorded by Ulysses. They show that there is a global anticorrelation between the solar wind bulk speed and the parameter κ value, which supports the kinetic theory that the solar wind acceleration is affected by the kinetic electrons, the suprathermal electrons. Fitting solar wind particle VDF as measured by CLUSTER has also been done using the generalized Kappa function.

The ratio of strahl electrons decreases and halo electrons grow with radial distance as seen in the low-latitude solar wind by Helios, Cluster, and Ulysses, as well as in the radial development of nonthermal electron populations. Observations of electron suprathermal tails in the solar wind provide evidence that these features may be present in the solar corona, where their mean free path is about 1 AU. There was a correlation between the ion charge measurements reported by Ulysses and the coronal Kappa VDF of electrons with a kappa index ranging from 5 to 10.

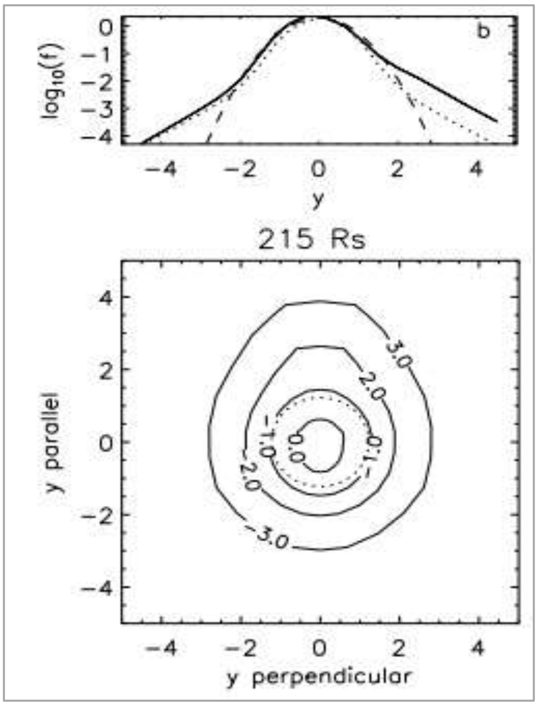

Figure 1: Electron Velocity Distribution Function in High-Speed Solar Wind Observed by the WIND Spacecraft at 1 AU

Using in situ Ulysses/URAP radio data in the solar wind, quasithermal noise spectroscopy with Kappa distributions was applied to quantify the suprathermal electron properties in space plasmas. The quasi-thermal electron fluctuations and the Doppler-shifted ion thermal fluctuations are responsible for this noise. Although a combination of two Maxwellians has been widely employed, the data reveal that a kappa distribution function better describes the electrons.

The distribution functions of solar wind ions (20Ne, 16O and 4He) as recorded by WIND and averaged over many days have also been fitted using Kappa functions. The existence of significant suprathermal tails causes low κ values (ranging from 2.4 to 4.7) to be achieved. At 1 AU, WIND detected corotating interaction region (CIR) events, which allowed for the measurement of suprathermal solar wind particles in the He+, He++, and H+ distribution functions.

IV. FORMATION MECHANISMS OF KAPPA DISTRIBUTIONS IN SPACE PLASMAS

The presence of Kappa-like distributions in space plasmas such as the solar wind and the corona, as well as the suprathermal tails of VDFs, have been the subject of several suggested processes. Hasegawa et al. (1985) suggested the first one by demonstrating that photon-induced Coulomb-field fluctuations amplify velocity-space diffusion in plasma bathed in a suprathermal radiation field. At all times, a power-law distribution is generated by this improved diffusion. The path lengths of the random walk hops in velocity space used by Collier (1993) to create distribution functions similar to Kappa are controlled by a power or L'evy flight probability distribution. Since the energy in space plasmas is not constant, as demonstrated by the same author, the maximal entropy is irrelevant.

According to Treumann's kinetic theory, a specific thermodynamic equilibrium state is corresponding to Kappa-like VDFs. The inclusion of correlations in a new Boltzmann-like collision term in kinetic theory was suggested. At thermal equilibrium, it produces a one-particle distribution function that is similar to but not exactly the Kappa distribution. This is because, in nonextensive Tsallis statistics, the entropy generalization can lead to distributions that resemble the Kappa distribution; these distributions are physically related to the long-range nature of the Coulomb potential, turbulence, and intermittency. Maximizing the Tsallis entropy yields the q distribution function, which is identical to the Kappa distribution. Non-Maxwellian distributions have a basic relationship to systems that are impacted by long-range interactions and correlations. In generalized thermostatics, core-halo distribution functions naturally represent an equilibrium state. A robust theoretical foundation for characterizing complex systems is provided by Kappa distributions, which emerge inherently from Tsallis statistical mechanics.

Supra thermal electrons can be produced by resonant contact with whistler waves in the solar corona and wind. Their findings demonstrate that anti sunward-propagating whistler waves can substantially affect the solar wind electron VDFs, resulting in the formation of halo and strahl populations as well as a more isotropic distribution at higher energies. This is achieved by incorporating these waves into a kinetic model to provide diffusion.

The cyclotron resonance and the magnetic Landau resonance of the linear waves allow plasma charges to accumulate energy in an ambient quasi-static magnetic field. For example, in Earth's foreshock, high-frequency whistler mode can increase electron energy, while in solar flares in the inner magnetosphere, MHD waves can accelerate electrons and protons alike.

Plasma particles can be energized by the nonlinear Landau damping in the presence of waves with enormous amplitudes. Plasma particle stochastic acceleration in compressional turbulence appears to agree with power law spectra in the heliosheath downstream of the solar wind termination shock. The discovery of finite-amplitude, low-frequency, obliquely propagating electromagnetic waves in the higher portions of the solar atmosphere with suprathermal tails in the presence of collisional damping was proposed as an explanation for this phenomenon. Additionally, heat fluxes, the existence of temperature anisotropies, and super diffusion processes can cause VDFs to exhibit nonthermal properties.

In a similar vein, Fokker-Planck (FP) equation solution and found a quasi-linear wave particle diffusion coefficient with a Kappa distribution that acts inversely on particle speed at velocities greater than the thermal speed. The relative intensities of wave-particle interactions and Coulomb collisions were investigated using the same FP equation. Additionally, Lie-Svendsen et al. (1997) used an FP model to study how high-energy tails emerged in the electron VDF. Be aware that under solar wind circumstances, a one-dimensional electrostatic Vlasov model has been suggested for the production of supra thermal electron tails. The issue of low-frequency waves and instabilities in uniform magnetized plasmas with bi-Maxwellian distribution was further used to show the potential evolution of Kappa velocity distribution.

In order to extend the velocity distributions to the observed power law functions, the kappa function is a valuable mathematical tool. This tool corresponds to the unique Maxwellian VDF that is associated with the value of κ as it approaches infinity, regardless of the methods by which suprathermal tails are formed.

V. THERMODYNAMICS OF KAPPA DISTRIBUTIONS

A common misunderstanding is that temperature is a basic independent characteristic and not something that depends on the kappa index. In physics, the idea of temperature is well-established in relation to particle systems in thermal equilibrium. This stems from the fact that the two primary conceptions of temperature—the kinetic conception put out by Maxwell in 1866 and the thermodynamic conception put forth by Clausius in 1862—are equivalent. We find the same equivalence for particle systems in stationary states outside of classical thermal equilibrium, where the Maxwell-Boltzmann behavior cannot characterize the particle distribution functions of velocities/energies. The particle distribution function can be broadly characterized by kappa distributions or combinations thereof when the system is in any arbitrarily given stationary state with a temperature.

There is a one-to-one correspondence between two thermodynamic systems that are in thermal equilibrium with one another, as stated in the zeroth law of thermodynamics. This is the same as two systems being in perfect thermal balance with one another. In fact, two systems are said to be in thermal equilibrium if (a) they are connected by a wall that allows only the flow of heat (energy) and (b) they remain constant throughout time. "Thermalization" describes a system of particles in a temperature-dependent stationary state, where the velocities and energy of the particles may be stabilized into a Maxwell-Boltzmann distribution or, more generally, a kappa distribution.

The mean particle kinetic energy is given by $U = \frac{1}{2} d k_B T$. Then, the kinetic definition is given by $T_{kin} = [2/(dk_B)].U$. However, the concept of "physical temperature" provides the thermodynamic definition of Clausius, $T_{phys} \equiv (\delta S/\delta U)^{-1}.[1 - \frac{1}{\kappa}.S/k_B$, that agrees with the zeroth law of thermodynamics; here, S stands for the system's entropy and U for its internal mean energy.

Using the generalized zeroth rule, we can get the physical temperature Tphys, which also acts as the kinetic definition of temperature in the context of non-extensive Statistical Mechanics. Thus, Tphys is the one to whom we can attribute all the benefits of a kinetically determined temperature, as opposed to other configurational versions. Since the generalized zeroth law is established at the genesis of Tphys, certain problems with respect to the kinetic formulation of the zeroth law of thermodynamics are totally remedied.

In the same way that mixing classical gases, which follow simple calorimetry rules, produces a mass-weighted average temperature of the combined plasma, mixing plasmas in two different stationary states (different kappa indices) produces the same result (Figure 2). To fix this, we showed that for kappa distributions of particles not in equilibrium, the two most fundamental definitions of temperature, (1) kinetic and (2) thermodynamic, are equivalent.

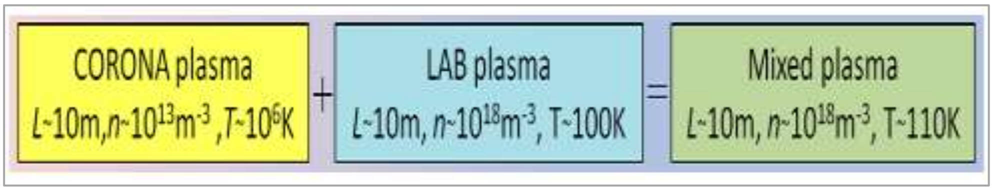

Figure 2: Space and Laboratory Plasmas Can be Understood Under the Same Unique Thermodynamics

One may use the kinetic definition to determine the temperature of the solar corona, and one can use a thermometer, which uses the thermodynamic definition, to measure the temperature of laboratory plasma. So, the resultant temperature is computed according to the principles of elementary calorimetry.

Having shown that temperature is independent of all other thermodynamic factors, the next step is to appropriately interpret the data showing connections between temperature and the kappa index. As an analogy, plasmas in classical thermal equilibrium can provide the following: Particles in plasma are characterized by Maxwell distributions that are conditional on density n and temperature T. Although n and T are two separate parameters, the streamlines of a plasma flow often exhibit a polytropic relation, T=A•na-1, that describes a variety of correlations between the two. What distinguishes one streamline from another is its polytropic index a, which in turn has a unique constant A.

The generalized polytropic relation T=f(n) has alternative definitions. For space plasmas that are not in thermal equilibrium, the kappa index (expressed as T=f(n,κ)) may also be a part of generalized polytropic relations. It is true that the relationship between κ and T is frequently given more attention than the dependency on density. Earth, Jupiter, and Saturn's magnetospheres have a positively associated kappa index with temperature values in their inner heliosheaths. Similar to how the observed links between density and temperature do not demonstrate a universal dependency between those two thermodynamic parameters, the observed correlations between kappa index and temperature do not indicate any such dependence. One must exercise caution in equating local interactions based on certain plasma streamlines or the entire plasma with any universal behavior pointing to the basis of physical laws, should one err.

Hence, finding the stationary state in classical thermal equilibrium by measuring the internal energy U does not reveal any unique traits. The Relativity Principle for Statistical Mechanics asserts that a system's internal energy may be described by any stationary state that can obtain it. This means that every state can be considered to have an identical description of the system's internal energy.

VI. CONCLUSION

The significance of kappa distributions in describing the nonequilibrium properties of space plasmas has been thoroughly investigated in this study. The presence of suprathermal particles may be described by the power-law tails of the kappa distribution, which are at high energy, in contrast to the traditional Maxwellian distributions. The evolution of such distributions in different spatial contexts is influenced by several physical phenomena, including wave-particle interactions, turbulent diffusion, stochastic acceleration, and nonextensive statistical effects. In order to avoid making errors in plasma diagnostics, particularly when calculating temperatures and energy contents, it is crucial to accurately calculate the kappa index, as this parameter influences the departure of the thermal equilibrium, as has been shown analytically. Furthermore, the research establishes thermodynamic consistency for kappa-distributed systems by demonstrating the equality of kinetic and generalized thermodynamic temperature definitions, which lends credence to their practical importance in stationary non-equilibrium situations. Additionally, it is noted that any correlations between plasma characteristics, such as temperature and kappa index, should not be taken for a generic dependency but rather as a property of a specific plasma state. Another proof of kappa distributions' use as modeling tools is that they may be used to represent Maxwellian behavior in the large kappa limit.

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